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tetrahedron are coplanar is a cubic surface, S, through the edges of the tetrahedron and having the vertices of the tetrahedron as nodes.[1]—The surfaces S, for the 15 tetrahedra determined by any six planes, meet in a point P (corresponding to the Wallace point above), and the pedal planes of P for the 15 tetrahedra are coincident.—Steiner remarked, in 1845, that S is the locus of the centers of the hyperboloids for which, when their equations are in standard form,

$$(1/a^2) + (1/b^2) = 1/c^2,$$

and for which a given tetrahedron is self-polar.

Among many discussions of the surfaces, S, the following may be mentioned: by Geiser, in *Journal für die reine und angewandte Mathematik*, vol. 69, 1868, p. 199 f.; by F. E. Eckhardt, in *Mathematische Annalen*, vol. 5, 1872, pp. 30–49; by E. Janhke, *Archiv der Mathematik* (Grunert), third series, vol. 4, 1903, especially pp. 275–276; by J. Neuberg, *Archiv der Mathematik* (Grunert), third series, vol. 16, 1910, p. 18 f.; and by W. H. Salmon, in *Archiv der Mathematik* (Grunert), third series, vol. 18, 1911, pp. 154–164. ARC.

SOLUTIONS.

2719 [1918, 302]. Proposed by R. P. BAKER, University of Iowa.

Show that[2] $2x(\log x)^2 - x(x-1)(x+3)\log x + (x-1)^2(3x-1)$ is negative for $1 < x < \infty$.

SOLUTION BY OTTO DUNKEL, Washington University.

Denoting the given expression by $f(x)$, its first two derivatives may be written as follows:

$$f'(x) = 2(\log x)^2 - (x-1)(3x+7)\log x + 8(x-1)^2,$$

$$f''(x) = \frac{3x^2+2x-2}{x}\varphi(x), \qquad \varphi(x) = \frac{(13x-7)(x-1)}{3x^2+2x-2} - 2\log x.$$

It will be observed that $f(1) = f'(1) = f''(1) = 0$. It will be shown that $f''(x)$ is negative for $x > 1$ and it will then follow that $f'(x)$ is also negative, and hence $f(x)$ is likewise negative for $x > 1$. The derivative of $\varphi(x)$ may be written

$$\varphi'(x) = -\frac{2(x-1)^3(9x-4)}{x(3x^2+2x-2)^2}$$

and it is clearly negative for $x > 1$. Since $\varphi(1) = 0$, it follows that $\varphi(x) < 0$. The first factor of $f''(x)$, which may be denoted by $\psi(x)$, has the roots $-(\sqrt{7}+1)/3$ and $x_1 = (\sqrt{7}-1)/3 = .549$, and hence $\psi(x) > 0$ for $x > 1$. Therefore, $f''(x) < 0$ for $x > 1$ and the desired result is proved.

The form of $\varphi'(x)$ shows that the first of the derivatives of $f(x)$ which do not vanish for $x = 1$ is $f^{\text{vi}}(x)$. The expressions above give at once $f^{\text{vi}}(1) = -20$ and this shows that $f(x)$ is also negative for values of $x < 1$. It will now be shown that the interval for such values extends down to $x = 0$. In the interval $0 \leqq x < 1$ the derivative $\varphi'(x)$ vanishes only for $4/9$ and it is negative before and positive after this value. Hence, $\varphi(x)$ has a minimum at this point. It will be found that $\varphi(4/9) = .312$, and so $\varphi(x)$ is positive from 0 to x_1. It is negative from x_1 to 1, since it vanishes at $x = 1$ and $\varphi'(x)$ is positive at all other points of this interval. On the other hand $\psi(x)$ is negative from 0 to x_1 and positive from x_1 to 1. At x_1 the product $\psi(x)\varphi(x)$ has a finite negative value and hence $f''(x)$ is negative from 0 to 1. From this follows that $f'(x)$ is positive and $f(x)$ is negative in the interval $0 \leqq x < 1$. Thus $f(x)$ is negative at every point of the interval $0 \leqq x < \infty$ except at the point $x = 1$ where it vanishes.

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[1] In *Annales de Mathématiques* (Gergonne), April, 1814, vol. 4, p. 320, the following problem was proposed (by Gergonne?) for solution: "The feet of the perpendiculars dropped on the faces of a tetrahedron from a point on the circumscribed sphere are coplanar." The incorrectness of this statement was proved by Durrande, in the same periodical, February, 1817, vol. 7, p. 255.

[2] It should be noted that this relation holds for "Naperian" but *not* for common logarithms. —EDITORS.